JAP-003-2011008 Seat No. ________

B. Sc. (Sem. I) (CBCS) Examination

November - 2019

MATH - 01 (A) : Calculus

Faculty Code : 003

Subject Code : 2011008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figure to the right indicate full marks of the question.

1 (A) Answer the following : **4**

(1) State the Lagrange's Mean Value Theorem.

(2) Find the value of c in Roll's Theorem for the function $f(x) = \sin 2x$ in $[0, \pi/2]$.

(3) Why Roll's Theorem is not applicable to the function $f(x) = e^{-2x}, x \in [-2, 2]$?

(4) True or False : Cauchy's Mean Value Theorem is a special case of Roll's Theorem.

(B) Attempt any **one** : **2**

(1) Express $f(x) = 2x^3 + 3x^2 - 8x + 7$ in term of $(x-2)$.

(2) Expand log x in powers of $x-1$ up to first three non-zero terms.

(C) Attempt any **one** : **3**

(1) Show that $\dfrac{\sin x - \sin y}{\cos y - \cos x} = \cot c$, where $0 < x < c < y < \dfrac{\pi}{2}$.

(2) Verify Lagrange's Mean Value Theorem for

$$f(x) = x + \frac{1}{x} \text{ in } \left[\frac{1}{2}, 3\right].$$

(D) Attempt any **one** : **5**

(1) Take $f(x) = \sin^{-1} x$ defined in $[a, b]$, where $0 < a < b < 1$. Then show that :

$$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$$

and hence deduce that

$$\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}.$$

(2) State and prove Lagrange's Mean Value Theorem.

2 (A) Answer the following : **4**

(1) What is the degree and order of the differential equation $(y'')^2 - \left[(y')^2 + 1\right]^3 = 0$?

(2) Define : Homogeneous function.

(3) True or False : $y' = \dfrac{\sqrt{y} + \sqrt{x+y}}{2\sqrt{x}}$ is a homogeneous differential equation.

(4) Define : Variable Separable differential equation.

(B) Attempt any **one** : **2**

(1) Find : $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

(2) Write general form and general solution of first order first degree linear differential equation.

(C) Attempt any **one** : **3**

(1) Evaluate : $\lim_{x \to \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2ex}$.

(2) Solve : $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$.

(D) Attempt any **one** : **5**

(1) Solve : $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

(2) If $\lim_{x \to 0} \frac{\sin 2x + p \sin x}{x^3}$ is finite, then find the value of p and hence evaluate the limit.

3 (A) Answer the following : **4**

(1) Define : Bernoulli's differential equation.

(2) Solve : $y = (x-1)p - p^2$, where $p = \frac{dy}{dx}$.

(3) State the condition for the equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact.

(4) Find an integrating factor of the differential equation $ydx - xdy = 0$.

(B) Attempt any **one** : **2**

(1) Solve : $p = \log(px - y)$, where $p = \frac{dy}{dx}$.

(2) Solve : $p^2 - 11p + 24 = 0$, where $p = \frac{dy}{dx}$.

(C) Attempt any **one** : **3**

(1) Solve : $x\frac{dy}{dx} + y\log y = xye^x$.

(2) Solve : $(2x\log x - xy)\ dy + 2y\ dy = 0$.

(D) Attempt any **one** : **5**

(1) Solve : $y + xp = x^4p^2$, where $p = \frac{dy}{dx}$.

(2) Solve : $\left(1 + e^{x/y}\right) + e^{x/y}\left(1 - \frac{x}{y}\right)\frac{dy}{dx} = 0$.

4 (A) Answer the following : **4**

(1) If roots of auxiliary equation are $-1, \pm i$, then write corresponding C.F.

(2) Find $D^3\left(\sin\frac{x}{2}\right)$, where $D = \frac{d}{dx}$.

(3) Find $\frac{1}{D^2}(\sin 2x)$, where $D = \frac{d}{dx}$.

(4) $\frac{1}{D-3}X =$ __________, where $D = \frac{d}{dx}$.

(B) Attempt any **one** : **2**

(1) Solve : $D^2y + 6Dy + 9y = 0$, where $D = \frac{d}{dx}$.

(2) Find the P.I. of $\left(D^2 + 9\right)y = \cos 2x$, where $D = \frac{d}{dx}$.

(C) Attempt any **one** : **3**

(1) Solve : $\left(4D^2 - 4D + 1\right)y = e^{x/2}$, where $D = \frac{d}{dx}$.

(2) In usual notation prove that :

$$\frac{1}{D-a}V(x) = e^{ax}\int e^{-ax}V(x)\,dx + c.$$

(D) Attempt any **one** : **5**

(1) Solve : $\left(D^2 + D\right)y = x^2 + 2x + 4$, where $D = \frac{d}{dx}$.

(2) Prove that :

$$\frac{1}{\varphi\left(D^2\right)}\sin(ax+b) = \begin{cases} \frac{1}{\varphi\left(-a^2\right)}\sin(ax+b) & \text{if } \varphi\left(-a^2\right) \neq 0, \\ \frac{x}{\varphi'\left(-a^2\right)}\sin(ax+b) & \text{if } \varphi\left(-a^2\right) = 0, \varphi'\left(-a^2\right) \neq 0. \end{cases}$$

5 (A) Answer the following : **4**

(1) Write general form of the Legendry's homogeneous linear differential equations.

(2) Reduce $\left(x^2D^2 + xD - 1\right)y = x^3$, where $D = \frac{d}{dx}$ in to differential equation with constant coefficient by taking $x = e^z$.

(3) True or False : Legendry's homogeneous linear differential equations is a special case of Cauchy's homogeneous linear differential equations.

(4) True or False : Cauchy's homogeneous linear differential equations is a special case of Legendry's homogeneous linear differential equations.

(B) Attempt any **one** : **2**

(1) Solve : $\left(4x^2D^2 + 16xD + 9\right)y = 0$, where $D = \frac{d}{dx}$.

(2) Solve : $\left(x^2D^2 + xD + 1\right)y = 0$, where $D = \frac{d}{dx}$.

(C) Attempt any **one** : **3**

(1) Solve : $\left(D + \frac{1}{x}\right)^2 y = \frac{1}{x^4}$, where $D = \frac{d}{dx}$.

(2) Solve : $\left(x^2D^2 - 2\right)y = x^2 + \frac{1}{x}$, where $D = \frac{d}{dx}$.

(D) Attempt any **one** : **5**

(1) Solve :

$$\left((1+x)^2 D^2 + (1+x) D + 1\right) y = 2\sin\left(\log(1+x)\right),$$

where $D = \dfrac{d}{dx}$.

(2) Solve :

$$\left((3x+2)^2 D^2 + 3(3x+2) D - 36\right) y = 3x^2 + 4x + 1,$$

where $D = \dfrac{d}{dx}$.
